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Nonsingular dilaton cosmology

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Abstract: We study spatially homogeneous and isotropic solutions to the equations of motion derived from dilaton gravity, in the presence of a special combination of higher derivative terms in the gravitational action. All solutions are nonsingular. For initial conditions resembling those in the pre-big-bang scenario, there are solutions corresponding to a spatially flat, bouncing Universe originating in a dilaton-dominated contracting phase and emerging as an expanding Friedmann Universe.

Keywords: Classical Theories of Gravity, Physics of the Early Universe.

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Contents

1. Introduction

The initial singularity is one of the outstanding problems of current cosmological models. In standard big bang cosmology, the existence of the initial singularity is an inevitable consequence of the Penrose-Hawking theorems [1, 2]. While scalar field-driven inflationary models such as chaotic inflation [3] resolve many of the problems faced by conventional cosmology, initial singularities are still generic, even when stochastic effects are included [4]. It is generally hoped that string theory may lead to a resolution of this problem. In an attempt to address the potential of string theory to remove the initial cosmological singularity, Gasperini and Veneziano initiated a program known as pre-big-bang cosmology [5] based on the low energy effective action resulting from string theory. At lowest order, this is the action of dilaton gravity,

$$S = -\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{R - \frac{1}{2}(\nabla\phi)^2 + \cdots\right\}, \tag{1.1}$$

where ϕ is the dilaton, $\kappa^2 = 8\pi G = 8\pi m_{\rm pl}^{-2}$, with G being the (4 dimensional) gravitational coupling, and $m_{\rm pl}$ the Planck mass. The field equations of pre-big-bang cosmology exhibit a new symmetry, scale factor duality, which (in the Einstein frame) maps an expanding Friedmann-Robertson-Walker (FRW) cosmology to a dilaton-dominated contracting inflationary phase. This raises the hope that it is possible to realize a nonsingular cosmology in which the Universe starts out in a cold dilaton-dominated contracting phase, goes through a bounce and then emerges as an expanding FRW Universe.[1]

[1]See ref. [6] for a recent review of pre-big-bang cosmology.

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Unfortunately, it has been shown that the two branches of pre-big-bang cosmology cannot be smoothly connected within the tree-level action [7, 8, 9, 10]. The contracting dilaton-dominated branch has a future singularity, whereas the expanding branch emerges from a past singularity. One-loop effects in superstring cosmology can regulate the singularity [11] and smoothly connect a contracting phase to an expanding phase, at least in the presence of spatial curvature. References [12]–[27] describe other attempts to regulate the singularities of pre-big-bang cosmology.

A natural approach to resolving the singularity problem of general relativity is to consider an effective theory of gravity which contains higher order terms, in addition to the Ricci scalar of the Einstein action. This approach is well motivated, since we expect that any effective action for classical gravity obtained from string theory, quantum gravity, or by integrating out matter fields, will contain higher derivative terms. Lastly, in the case of string theory, t-duality provides further evidence that physical quantities will remain finite at all times [28]. Thus, it is extremely natural to consider higher derivative effective gravity theories when studying the properties of space-time at large curvatures.

The various possible extensions to classical general relativity all hold out the hope of a fully nonsingular theory of gravity, and therefore a nonsingular cosmology. However, this promise has yet to be realized in an effective theory of gravity that is rigorously derived from a well motivated model of Planck-scale physics. In this paper we approach the problem from a different perspective, and ask instead whether it is possible to derive a higher order theory of gravity which will produce a cosmology that captures the qualitative features of the nonsingular cosmological evolution envisaged by the pre-big-bang scenario.

An approach to explicitly constructing an effective gravitational action which insures that physical invariants are always finite is given in refs. [29, 30]. The resulting action includes a particular combination of quadratic invariants of the Riemann tensor added to the usual Einstein-Hilbert action for gravity. This term forces all solutions of the equations of motion to approach de Sitter space-time at high curvature, and therefore renders them nonsingular. The model thus obtained is a specific higher-derivative gravity theory.

We should mention that a further difficulty with a perturbative approach to the cosmological singularity problem is that the perturbation expansion is expected to break down at energy scales lower than the scale at which the singularity will be smeared out. References [29, 30] also address the problem of ensuring that there is maximum allowable curvature in a given cosmological model, using a similar technique to the one employed to eliminate singularities. At this point, however, we have chosen to focus our attention on the singularity problem alone, and have not imposed a limit on the curvature, beyond that implied by the removal of singularities which ensures that for any given solution the curvature will be bounded.

The simplest way of achieving a nonsingular cosmology is to add an invariant I_2

to the action, with the property that $I_2 = 0$ is true if and only if the spacetime is a de Sitter space. By coupling I_2 into the gravitational action via a Lagrange multiplier field ψ with a potential chosen to ensure that $I_2 \to 0$ at large curvatures we can require that all solutions approach de Sitter space at large curvature, thereby removing the singularity. For homogeneous and isotropic spacetimes, a choice for I_2 which satisfies this condition is

$$I_2 = \sqrt{4R_{\mu\nu}R^{\mu\nu} - R^2}\,. \tag{1.2}$$

The simplest way of writing the resulting action is

$$S(g_{\mu\nu}, \psi) = \int d^4x\sqrt{-g}(R + \psi I_2 + V(\psi))\,, \tag{1.3}$$

where $V(\psi)$ is a function chosen such that the action has the correct (for $\psi \to 0$) Einsteinian low curvature limit, whereas for $|\psi| \to \infty$ the constraint equation forces $I_2 \to 0$.

In this paper, we investigate the consequences of adding the same higher derivative terms to the action of pre-big-bang cosmology, and examine whether it can eliminate the singularities and produce a smooth bounce connecting a contracting phase to an expanding Universe. Our main result is that it is possible to find a potential $V(\psi)$ for the Lagrange multipliers which ensures that all cosmological solutions of the extended action for dilaton gravity are nonsingular. Moreover, we show that there is a class of solutions corresponding to a contracting Universe smoothly connected to an expanding FRW phase via a bounce. Furthermore, this happens even in the absence of spatial curvature. Our model, therefore, constitutes a successful implementation of the goals of pre-big-bang cosmology.

2. Action and equations of motion

Our starting point is the action (1.1) for dilaton gravity (written in the Einstein frame) to which we add the higher derivative term given by I_2, in analogy to what was done in the absence of the dilaton in refs. [29, 30]:

$$S = \frac{-1}{2\kappa^2}\int d^4x\sqrt{-g}\left\{R - \frac{1}{2}(\nabla\phi)^2 + c\psi e^{\gamma\phi}I_2 + V(\psi)\right\}. \tag{2.1}$$

For the moment, we allow a general coupling between I_2 and the dilaton. Minimal coupling corresponds to setting the constant γ equal to zero. The constant c rescales the Lagrange multiplier field ψ, and will be chosen to simplify the equations of motion.

Restricted to a homogeneous and isotropic metric of the form

$$ds^2 = dt^2 - a(t)^2(\frac{1}{1-kr^2}dr^2 + r^2 d\Omega^2)\,, \tag{2.2}$$

where $d\Omega^2$ is the metric on S^2, the equations of motion resulting from (2.1) become

$$\ddot{\phi} + 3H\dot{\phi} + \gamma c\psi e^{\gamma\phi}\sqrt{12}\left(\frac{k}{a^2} - \dot{H}\right) = 0\,,$$

$$
\begin{aligned}
\dot{H} &= \frac{k}{a^2} - \frac{e^{-\gamma\phi}}{c\sqrt{12}}\frac{\partial V}{\partial \psi}\,, \\
6\frac{k}{a^2} + 6H^2 - \frac{\dot{\phi}^2}{2} - V(\psi) &= \\
&= c e^{\gamma\phi}\sqrt{12}\left(3H^2\psi - \frac{k}{a^2}\psi + H(\dot{\psi} + \gamma\dot{\phi}\psi)\right)\,, \qquad (2.3)
\end{aligned}
$$

where dots denote derivatives with respect to time, t.

Since our initial goal is to construct a spatially flat, bouncing Universe, we set the curvature constant $k = 0$. We will, for the moment, consider minimal coupling of ϕ to I_2, which fixes $\gamma = 0$. To eliminate useless constant coefficients in the equations of motion, it is convenient to choose $c\sqrt{12} = 1$. The resulting equations of motion become

$$
\begin{aligned}
\dot{\psi} &= -3H\psi + 6H - \frac{1}{H}(\frac{1}{2}\chi^2 + V(\psi)), \\
\dot{H} &= -V'(\psi), \\
\dot{\chi} &= -3H\chi, \qquad (2.4)
\end{aligned}
$$

with $\chi = \dot{\phi}$ and a prime ($\prime$) signifying derivatives with respect to ψ.

We next turn to a discussion of the criteria which the potential $V(\psi)$ must satisfy. At small curvatures, the terms in the action (2.1) which depend on ψ must be negligible compared to the usual terms of dilaton gravity. This is ensured by demanding

$$
V(\psi) \sim \psi^2 \qquad |\psi| \to 0\,, \qquad (2.5)
$$

as the region of small $|\psi|$ will correspond to the low curvature domain [30]. In order to implement the limiting curvature hypothesis, the invariant I_2 must tend to zero, and thus the metric $g_{\mu\nu}$ will tend to a de Sitter metric at large curvatures, i.e. for $|\psi| \to \infty$. From the variational equation with respect to ψ, it is obvious that this requires

$$
V(\psi) \to \text{const} \qquad |\psi| \to \infty\,. \qquad (2.6)
$$

Conditions (2.5) and (2.6) are the same as those employed in refs. [29, 30], but they do not fully constrain the potential. In order to obtain a bouncing solution in the absence of spatial curvature it is necessary to add a third criterion: the equations must allow a configuration with $H = 0$ and $\psi \neq 0$. From the equation of motion for ψ in (2.4) it follows that $V(\psi)$ must become negative, assuming that it is positive for small $|\psi|$. Let ψ_b denote the nontrivial zero of $V(\psi)$:

$$
V(\psi_b) = 0\,. \qquad (2.7)
$$

In the absence of the dilaton, ψ_b will correspond to the value of ψ at the bounce. In the presence of ϕ, the value of $|\psi|$ at the bounce will depend on χ and will be larger than $|\psi_b|$.

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A simple potential which satisfies the conditions (2.5), (2.6) and (2.7) is

$$V(\psi) = \frac{\psi^2 - \frac{1}{16}\psi^4}{1 + \frac{1}{32}\psi^4}. \tag{2.8}$$

Note that the potential used in refs. [29, 30] (which is slightly simpler) does not satisfy condition (2.7).

3. Phase diagram of solutions in the absence of the dilaton

The conditions, (2.5), (2.6), (2.7), on the potential, $V(\psi)$, discussed in the previous section are necessary but not sufficient to obtain a nonsingular cosmology. These conditions ensure that all solutions which approach large values of $|\psi|$ are nonsingular, but the possibility of geodesic incompleteness for solutions which always remain within the small $|\psi|$ region remains to be studied. In this section we determine the phase plane (ψ, H) of our model in the absence of the dilaton. We use analytical and numerical methods to study the trajectories of solutions of (2.4) in the phase plane and thus explicitly demonstrate the absence of singularities. Our model therefore yields a further example of a higher derivative gravity theory without cosmological singularities. In addition, and unlike the model of refs. [29, 30], we shall show that our theory admits spatially flat bouncing solutions.

There are several special points and curves on the phase plane (ψ, H). First, the point $(\psi, H) = (0, 0)$ corresponds to Minkowski space-time. The potential $V(\psi)$ vanishes at this point, but it also vanishes at the points

$$\psi_b = \pm 4\,. \tag{3.1}$$

As discussed earlier, the phase plane points $(\psi_b, 0)$ correspond to bouncing points of cosmological trajectories.

The derivative of $V(\psi)$, and hence $\dot{H}$, vanishes for the values

$$\psi_d = \pm 2\,. \tag{3.2}$$

The phase plane lines (ψ_d, H) are therefore lines along which $\dot{H} = 0$.

To demonstrate that the point $(\psi, H) = (4, 0)$ is in fact a bounce, we expand the ψ equation of motion near $H = 0$, which yields

$$H\dot{\psi} = -V\,. \tag{3.3}$$

Thus, as we cross the $H = 0$ axis, the sign of $\dot{\psi}$ changes. Contracting solutions with $2 < \psi < 4$ have $\dot{\psi} > 0$ and approach the point $(4, 0)$ in finite time since $\dot{H}$ is positive and does not tend to zero, and emerge for $H > 0$ as expanding trajectories with decreasing curvature (since $\dot{\psi} < 0$). The trajectories in the phase plane are symmetric about the $H = 0$ axis, except that the time arrows are reversed.

Next, we expand the equations near the origin of the phase plane and obtain

$$\frac{d\psi}{dH} \simeq \frac{1}{2H}(\psi - \frac{6H^2}{\psi}), \tag{3.4}$$

from which we see immediately the existence of critical lines located at

$$\psi_c(H) = \pm\sqrt{6}H\,. \tag{3.5}$$

Focusing on the contracting solutions, the equation of motion for $\dot{H}$ reduces to

$$\dot{H} \simeq -2\psi. \tag{3.6}$$

Consequently, trajectories which lie above the critical line have $\dot{H} < 0$ and (from eq. (3.4)) $\dot{\psi} > 0$. These trajectories thus are directed towards the line $\psi = 2$ where $\dot{H}$ changes sign. Provided they do not cross the critical line, solutions which start out in this region of the phase plane are thus candidates for spatially flat bouncing Universes.

Solutions below the critical line have $\dot{\psi} < 0$ and will hence not exhibit a bounce. Note that the critical line is not itself a trajectory of the dynamics. Indeed, on the critical line trajectories point in vertical direction, since $d\psi/dH = 0$. There is a separatrix line between phase plane trajectories which start near the origin and which reach $\psi = 2$ (and are thus candidates for a bouncing Universe) and those trajectories which cross the critical line $\psi_c(H)$ and turn around, i.e. become solutions with $\dot{\psi} < 0$. To determine the location of the separatrix line, we solve the equations (2.4) near the origin of the phase plane for $|H| \ll \psi$, in which case the variational equation with respect to ψ in (2.4) becomes

$$H\dot{\psi} \simeq -\psi^2\,, \tag{3.7}$$

which must be solved together with (3.6). After differentiating (3.6) with respect to time and substituting (3.7) to eliminate $\dot{\psi}$, we obtain the equation

$$H\ddot{H} - \frac{1}{2}\dot{H}^2 = 0\,, \tag{3.8}$$

which has the solution

$$H(t) = -ct^2\,, \tag{3.9}$$

where c is a constant which labels the different trajectories. The second solution of (3.8) is $H(t) = \text{const}$ and is thus uninteresting. From (3.6) it follows that for the above solution

$$\psi(t) = ct\,. \tag{3.10}$$

Equations (3.9) and (3.10) give the solutions of the dynamical equations near the origin of the phase plane, provided that $|H| \ll \psi$. To get an idea for where the separatrix line lies, we demand that the trajectory be above the critical line at $\psi = 2$. This condition becomes

$$|H| = \frac{1}{c}\psi^2 \qquad \text{with} c \geq 4\sqrt{2}\,. \tag{3.11}$$

Setting $c = 4\sqrt{2}$ in (3.11) gives a first estimate for the location of the separatrix line. Since the set of phase plane trajectories near the origin is labeled by the parameter c, it follows from (3.11) that the set of contracting Universes starting near Minkowski space-time which are candidates for a spatially flat bouncing Universe has finite measure.

When $\psi = \pm 2$, $\dot{H}$ changes sign. Contracting solutions which begin near the origin in the phase plane reach a maximal value of $|H|$ at $\psi = 2$. What happens to these trajectories next depends on the value of $|H|$ for $\psi = 2$. If

$$|H(\psi = 2)| \ll 1\,, \tag{3.12}$$

then (3.3) gives a good approximation to the dynamics in the region $2 < \psi < 4$, and we conclude that the trajectories bounce. Making use of $|H| = \psi^2/c$ (see (3.11)), the condition (3.12) becomes

$$c \gg 4\,, \tag{3.13}$$

which is consistent with the previous condition (3.11) for bouncing trajectories.

Trajectories with $|H(\psi = 2)| \gg 1$ also reach their maximal value of $|H|$ at $\psi = 2$, but they do not bounce because the ψ equation of motion can now be approximated by

$$\dot{\psi} \simeq -3H\psi\,, \tag{3.14}$$

which implies that ψ keeps growing indefinitely. In combination with the equation of motion for H, which yields that

$$\dot{H} \simeq 0 \qquad \text{for } |\psi| \gg 4\,, \tag{3.15}$$

we see that the solutions tend to contracting de Sitter ones. Solutions below the critical line have $\dot{\psi} < 0$ and will not lead to a bouncing Universe.

For values of ψ approaching $\psi = 2$, the two critical lines diverge to $|H| \to \infty$. This can be seen immediately from the ψ equation of motion in (2.4). For large values of $|H|$, the right hand side of this equation is dominated by the first two terms. In the absence of the third term, the solution of $\dot{\psi} = 0$ which determines the critical line would be $\psi = 2$. The third term, however, leads to a small but positive correction to the first term (in the bottom right quadrant of the phase plane which we are considering throughout), thus shifting the critical line slightly to the left of $\psi = 2$, satisfying

$$-3H\psi + 6H - \frac{1}{H}V = 0\,. \tag{3.16}$$

To prove that all solutions of the equations (2.4) are nonsingular, we must show that the solutions asymptotically tend to either the origin of the phase plane (Minkowski space-time), or to de Sitter space (the regions $|\psi| \to \infty$). Solutions which have $|H| \to \infty$ for finite $|\psi|$ must be excluded. The regions along the critical lines are the only part of the phase plane where there is the danger of singular trajectories. However,

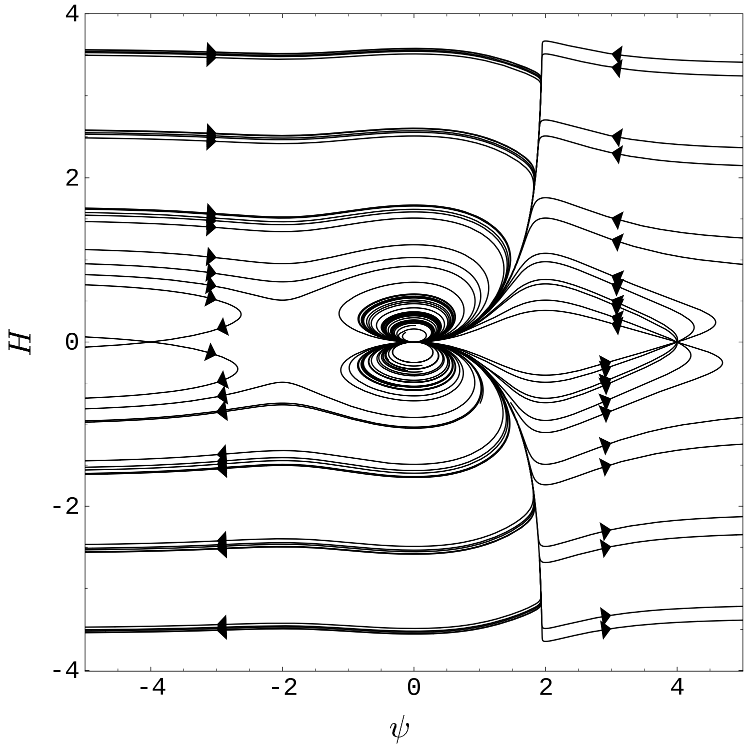

Figure 1: The phase portrait for solutions of the equations of motion, with the dilaton kinetic energy (χ) set to zero is shown. The Hubble parameter, H, is plotted on the vertical axis, while ψ is plotted on the horizontal.

differentiating the ψ equation of motion with respect to time, evaluating the result along the critical line and making use of (3.16), we find

$$\ddot{\psi} = -V' \frac{2V}{H^2} < 0 \,. \tag{3.17}$$

Hence, the critical line is not an attractor, but rather trajectories peel away from the line and tend to the asymptotic de Sitter region.

We have also solved the equations of motion (2.4) numerically. The resulting phase diagram is shown in figure 1. The absence of singular solutions is manifest. In the asymptotic regions $|\psi| \gg 4$, all solutions tend to de Sitter space. The critical lines are seen to repel contracting solutions towards the asymptotic de Sitter regions. The most interesting class of solutions are those corresponding to a cosmological bounce. As predicted, they form a set of finite measure among solutions which start out close to the origin of the phase plane. There are also solutions which “oscillate” about Minkowski space-time. These solutions are further discussed in the following section, since we expect that they will be strongly perturbed by the presence of matter, in particular by the dilaton.

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4. Effects of the dilaton on the phase space trajectories

In the presence of the dilaton, the phase space becomes three dimensional $(\psi(t), H(t), \chi(t))$ and therefore more difficult to discuss analytically. We first note that in the Einstein frame, the dilaton corresponds to a homogeneous free massless scalar field with equation of state

$$p = \rho\,, \tag{4.1}$$

where p and ρ denote pressure and energy density, respectively. This implies

$$\rho(t) \propto a(t)^{-6} \tag{4.2}$$

or $N = 6$ in the notation of [30]. Thus, it is already clear from the analysis of [30] that the dilaton will not introduce any singularities into the system. In fact, from the equation of motion for H (see (2.4)) it follows that, in the region of large $|H|$, the presence of χ will not change the phase space trajectories projected onto the (ψ, H) plane (a property called "asymptotic freedom" in [30]). However, the presence of χ will greatly accelerate the time evolution of ψ on the given (ψ, H) trajectory. This is easy to see for the contracting de Sitter solutions, since in this case the χ equation of motion (see (2.4) leads to exponential growth of χ which, inserted into the ψ equation of motion, demonstrates that at large values of $|\psi|$, the χ^2 term dominates the evolution of ψ.

We next study the effect of χ on the trajectories in the (ψ, H) phase plane. The role of the lines $\psi = \pm 2$ remains unchanged: they correspond to maxima of $|H|$ for any given trajectory. However, the condition for the bounce changes. Instead of $V(\psi_b) = 0$, it now follows from (2.4) that the condition becomes

$$\frac{1}{2}\chi^2 + V(\psi_b) = 0\,. \tag{4.3}$$

Hence, $|\psi_b|$ is shifted to a larger (and dilaton-dependent) value. Note that for very large values of $|\chi|$ for which the above equation has no solutions there will be no bounce.

The presence of χ also changes the critical lines. For a fixed value of ψ, the condition $\dot{\psi} = 0$ which determines the critical line occurs at a larger value of $|H|$ than it does in the absence of χ, as can easily be seen from the ψ equation of motion (see (2.4). In addition, for fixed initial (ψ, H) the value of $\dot{\psi}$ for contracting solutions is larger with $\chi \neq 0$ than with $\chi = 0$, so we conclude that adding a small value of χ increases the range of initial conditions in the (ψ, H) plane which lead to a bounce. Furthermore the larger the value of χ, the larger the effect is (as long as (4.3) still has a solution).

We will now argue that collapsing solutions in the presence of a small $|\chi|$ quite generically lead to a spatially flat bouncing Universe. Consider initial conditions with small but positive χ which in the (ψ, H) plane lie below the separatrix line and which initially "oscillate" about Minkowski space-time. Note that since

$$\ddot{H} = -V'\dot{\psi}\,, \tag{4.4}$$

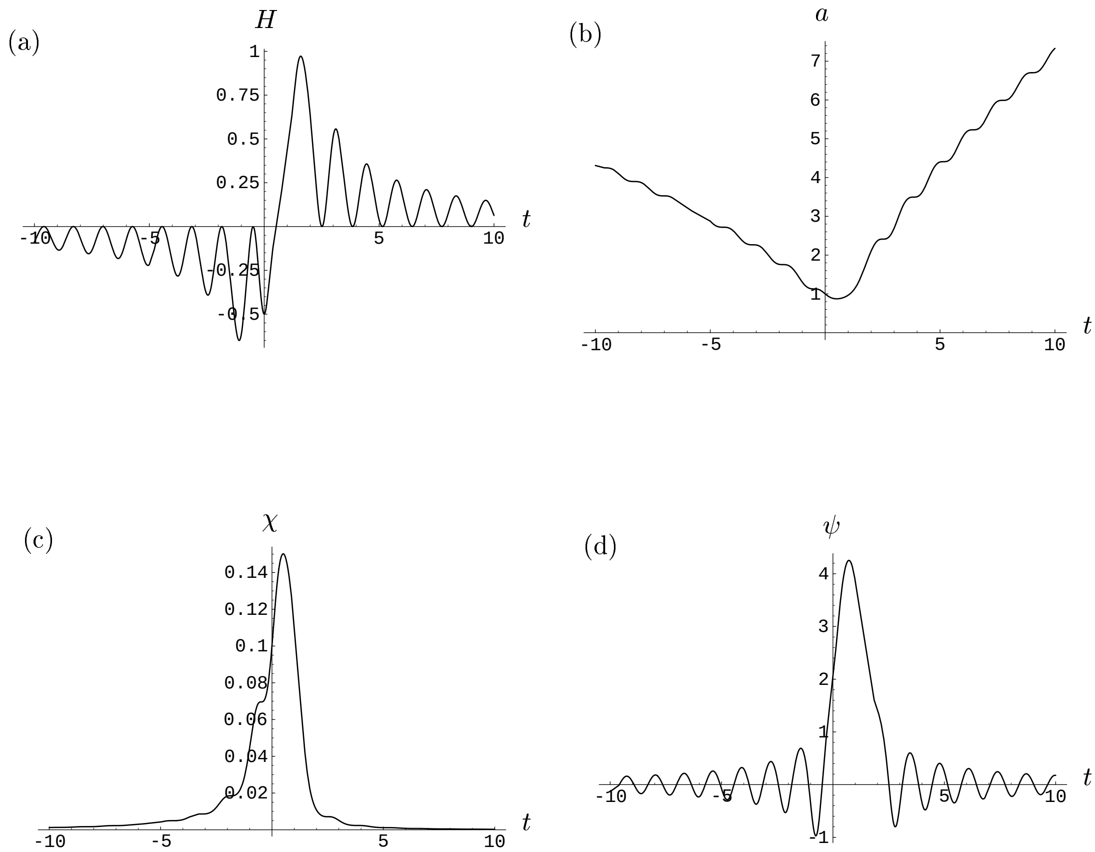

Figure 2: A specific bouncing solution with a non-trivial contribution from the dilaton is plotted. Panel a) shows the evolution of H as a function of time, Panel b) depicts that of $a(t)$, Panel c) that of $\chi(t)$ and Panel d) that of $\psi(t)$.

and since $\dot{\psi} > 0$ for these trajectories, they do not bounce upon reaching $H = 0$ but start another cycle with $H < 0$. Since $H \leq 0$ at all times, $\chi(t)$ is increasing. Eventually, $\chi(t)$ reaches a sufficiently large value such that the trajectory crosses the "separatrix" in the (ψ, H) plane and evolves past $\psi = 2$ to a successful bounce. An example of such a trajectory is shown in Figure 2.

It is now obviously possible to construct initial conditions in which the Universe is initially dominated by the dilaton and contracting towards a bounce. For example, we can take the initial conditions for (ψ, H, χ) to be those of the solution in Figure 2 at the end of the cycle preceding the bounce. We have thus shown that our model provides a successful implementation of the evolution postulated in pre-big-bang cosmology. Note that after the bounce, the dilaton rapidly becomes irrelevant to the evolution in the expanding phase. Note, in particular, that the dilaton tends to a constant. This feature is different from what happens in other attempts to achieve a branch change where the dilaton continues to grow in the expanding phase.

5. Conclusions

We have shown that the "limiting curvature construction" of refs. [29, 30] can be applied to dilaton cosmology to yield spatially flat bouncing cosmological solutions. In particular, this enables us to implement a model which successfully implement the qualitative evolution associated with a branch change in pre-big-bang cosmology.

Specifically, we have investigated spatially homogeneous, isotropic, and flat solutions of the equations of motion for dilaton gravity coupled to, I_2, a special combination of invariants quadratic in the Riemann curvature. The coupling is provided by a Lagrange multiplier field ψ. We have determined the conditions which must be imposed on the potential $V(\psi)$ required in order to guarantee that all solutions are singularity-free, and to permit bouncing scenarios.

We then studied the phase space of trajectories and demonstrated, both analytically and numerically, that all the solutions to the equations of motion which follow from our action are nonsingular. We identified a large class of contracting trajectories which start out near the origin in phase space and develop a cosmological bounce. The addition of the dilaton increases the fraction of phase space for which solutions bounce. These bouncing cosmologies display the qualitative behavior associated with the pre-big bang branch change.

An interesting result of our work is the conclusion that the dilaton is not necessary to obtain a spatially flat bouncing cosmology. Also, in many cases a dilaton-dominated bounce is preceded by a period when the dilaton has a negligible effect on the dynamics of $\psi(t)$ and $H(t)$.

Our work does not address the issue fixing the value of the dilaton at late times. Nor does it address recent objections [31, 32, 33] to the pre-big-bang scenario. In work in progress, we are investigating the possibility that our implementation of the branch change will alleviate some of these difficulties, in particular the flatness problem and the problem of how to obtain a large Universe. The relevant feature of our model which we expect to play an important role in addressing these problems is that the dynamics of our bounce are determined by the higher derivative gravity terms which lead to de Sitter phases for many trajectories. Thus, on balance, our model looks quite promising.

The obvious drawback of our method is that the extra terms in the action are put in by hand rather than being derived from fundamental physics. There are, however, several justifications for following our approach. First, it is generally expected that physical invariants must be limited in string theory. This should, at high curvatures, be reflected in the effective action for gravity, and our action is an easy way to realize limiting quantities. Secondly, higher derivative terms will inevitably arise in effective theories of gravity, such as those derived from string theory, from other approaches to quantum gravity, or from quantizing matter fields in curved space-time. Our action, even though not directly derived from string theory, demonstrates that a specific set of higher curvature corrections can both ensure that all relevant physical invariants remain finite, and produce a bouncing nonsingular cosmology.

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